

Queen Mary and Westfield College
UNIVERSITY OF LONDON
B.Sc. Examinations 2004

MAS101 Calculus I, Resit candidates only

Wednesday 5 May 2004, 2.30–4.30 pm

Duration: TWO HOURS

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

SECTION A

Each question carries 8 marks. You should attempt ALL 5 questions.

A1. Let $y = x^3 \ln(2x)$. Find $\frac{d^3y}{dx^3}$.

A2. Find all solutions in the complex numbers of the following equations:

(a) $z^2 + (3 + i)z + 4 + 3i = 0$.

(b) $z^2 + (3 - i)z + 4 - 3i = 0$.

(c) $z^2 - (3 + i)z + 4 + 3i = 0$.

A3. Draw a sketch of the complex plane (Argand diagram) including the following subsets:

- $\{z \in \mathbb{C}; z^3 = 8\}$.
- $\{z \in \mathbb{C}; |z - 1| > |z - 2i|\}$.

- A4. Solve the following differential equation, subject to the conditions $y(\pi/8) = y(-\pi/8) = \sqrt{2}$:

$$\frac{d^2y}{dx^2} + 4y = 0.$$

- A5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \int_0^{x^2} \sqrt{1 + \sin t} \, dt.$$

Find an expression for the derivative $f'(x)$.

SECTION B

Each question carries 20 marks. You may attempt all questions but, except for the award of a bare pass, only marks for the best 3 questions will be counted.

- B6. Find the general solution of the differential equation

$$\frac{dy}{dx} + 5y = \sinh 5x.$$

If y is a solution to this equation, does $\lim_{x \rightarrow -\infty} y(x)$ exist? Give reasons for your answer.

- B7. Compute the following Riemann integrals.

(a) $\int_0^\pi (\cos x)^2 \sin x \, dx.$

(b) $\int_0^1 x^2 e^{x/3} \, dx.$

(c) $\int_0^3 |x - 1| \, dx.$

- B8. (a) State the Mean Value Theorem for a differentiable function $f : \mathbb{R} \rightarrow \mathbb{R}$ and real numbers $a < b$.
(b) By (a) with $f = \sin$, or otherwise, show that $\sin 0.1 < 0.1$.
(c) Suppose f is the function x^2 . Show that the number c stated to exist in the Mean Value Theorem can be chosen as the average of a and b . Is the same true when f is the function x^3 and $a = 0$?

B9. (a) Write down the first three nonzero terms of the Taylor series for the following functions:

(i) $\sin x^2$ about $x = 0$.

(ii) $\frac{1}{2-3x}$ about $x = 0$.

(b) By using l'Hôpital's rule or otherwise, determine the following limits.

(i) $\lim_{x \rightarrow 0} \frac{x^4 + 3x^2}{2x^3 - x^2}$.

(ii) $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\sin 3x}$.

(iii) $\lim_{x \rightarrow 0} x^{-1} \ln(x+1)$.

B10. State De Moivre's Theorem. Writing $s = \sin x$ and $c = \cos x$, use De Moivre's Theorem to find expressions for $\cos 5x$ and $\sin 5x$ in terms of s and c . Hence express $\cos 5x$ as a polynomial in c , and use this polynomial to show that if $x = \pi/5$, then

$$16c^5 - 20c^3 + 5c + 1 = 0.$$

End of examination paper